Airborne Electromagnetic Sensing of Sea-Ice Thickness

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Summary

One can compute the terrain response for a rigid, frequency domain, airborne electromagnetic (AEM) system when it is flown over a perfectly conducting medium with arbitrary surface relief by numerically solving the corresponding Neumann boundary value problem. This approach can be used to accurately predict the observable secondary magnetic field over sea water covered with multi-year ice even when the sea water surface is indented by sea-ice keels. The results obtained for a number of keel models indicate that a conventional AEM system can be used to determine the variations in sea-ice thickness in areas with rough sea-ice topography.

Introduction

Becker, Morrison and Smits (1983) showed that the thickness of sea ice can be determined accurately with any rigid-boom AEM system if the sea-ice cover is uniform. Experience, however, shows that the thickness of sea ice varies rapidly with position and a significant problem in the mapping of sea-ice topography is the presence of submerged keels. These vary in depth and are variably spaced by a distance of the order of 50 - 100 m (Wadhams and Horne, 1980). In this context, it is evident that one-dimensional interpretation of AEM data taken over irregular sea-ice topography is erroneous and that two- or three-dimensional models are needed for its interpretation. To meet this requirement, we present a numerical solution for AEM system response over multi-dimensional sea-ice features under the assumption that the sea water can be regarded as infinitely conductive and the sea ice infinitely resistive. These assumptions dictate that the total magnetic field normal to the water-ice surface be null and define the Neumann boundary value problem, which can be solved by integral equation methods.

Solution of the Neumann problem

Consider an alternating magnetic dipole (current loop) source D, located in free space as shown in Figure 1. Its orientation is arbitrary and it is positioned over a homogeneous perfectly conductive medium with three-dimensional surface relief. This model simulates an AEM dipole transmitter flying over an extensive sea-ice cover which indents the sea water. The perfectly conductive medium represents the sea water and its surface constitutes the interface between the ice and the water. Effects of sea-ice conduction are neglected because of its comparatively high resistivity.

The transmitter induces electric currents on the surface of the sea water, which then give rise to the secondary magnetic field, $\mathbf{H}_s$ in free space. It is our objective to calculate this quantity at any point above the surface S.

Under the quasi-static approximation, we may relate $\mathbf{H}_s$ to a scalar magnetic potential ϕ.

$$\mathbf{H}_s=-\nabla\phi \qquad (1)$$

We also have $\nabla\cdot\mathbf{H}_s=0$ and correspondingly

$$\nabla^2\phi=0 \qquad (2)$$

Since the lower medium is assumed to be infinitely conductive, the normal component of the total magnetic field must vanish on the surface S, i.e.

$$H_{pn}+H_{sn}=0 \qquad on\ S \qquad (3)$$

Here, H_{sn} and H_{pn} are the normal components of the secondary and primary magnetic fields respectively. Hence,

$$\frac{\partial\phi}{\partial n}\mid_S = -H_{sn}\mid_S = H_{pn}\mid_S \qquad (4)$$

Equations (2) and (4) constitute the Neumann boundary value problem. Given the normal derivative of the potential on a surface S, we wish to calculate the potential itself in the free space. Once ϕ is found, $\mathbf{H}_s$ may be calculated from equation (1).

The solution of the exterior Neumann problem can be expressed as the potential of a surface charge layer (Graham, 1980)

$$\phi(O)=\int_S\frac{\xi(P)}{r_{OP}}ds \qquad (5)$$

where $\xi(P)$ is a fictitious single-layer charge function, r_{OP} is the distance between points O and P (see Figure 1). The fictitious charge function satisfies a Fredholm integral equation of the second kind

$$\xi(M)=-\frac{1}{2\pi}\frac{\partial\phi}{\partial n}\mid_S - \frac{1}{2\pi}\int_S\xi(P)\frac{cos(\mathbf{r}_{PM},\mathbf{N}_M)}{r_{PM}^2}ds \qquad (6)$$

where ($\mathbf{r}_{PM},\mathbf{N}_M$) is the angle between $\mathbf{r}_{PM}$ (the vector connecting P to M) and $\mathbf{N}_M$ (the unit normal vector at M). In our problem, the sea water surface S extends to infinity.

To solve the integral equation (6), We use the successive approximation method (Mikhlin, 1964). The initial solution is assumed to be $\xi(M)=\frac{1}{2\pi}H_{pn}$. We then use this value in the integral to compute an improved value of $\xi(M)$ and so on. Usually, the process converges fast and the calculation of the field is accurate to the third digit by using about four iterations. In the special case where the surface S forms a plane, the Neumann solution for a magnetic dipole source is identical to that obtained by the method of images (Jackson, 1975).

The accuracy of this numerical technique was checked by comparing the results with analytical data and scale model data. They show that the numerical results are accurate to about ± 5%.

Numerical models

To investigate the problem of determining the variation of sea-ice thickness by AEM methods, we have calculated the airborne electromagnetic response for a number of two-dimensional sea-ice keels defined below by equation (7) with different parameters A and r. For this model (cf. Figure 2), the cross section of the indented surface is a Gaussian distribution curve that simulates a smoothed ice keel. Its relief is given by

$$D(x)=A\ exp(-\frac{x^2}{2\tau^2}) \qquad (7)$$

where A is the drawdown of the surface and τ the deviation. The electromagnetic system has a coil separation of 6.5m and is " flown " 25m above the upper ice surface which is flat. With the exception of the zone containing the keel, the sea ice is 5m thick and is assumed to have negligible electrical conductivity. The data is calculated for a typical helicopter system (Morrison, Becker and Smits, 1983) for both the horizontal-axis-coaxial-coil configuration (Hx) and the vertical-axis-coplanar one (Hz). Because the theoretical calculations assume infinite conductivity for the sea water, the frequency of operation is not a factor here. From a practical view point however, we expect these computations to be valid at frequencies greater than 30kHz where the skin depth in sea water is less than 1.5m.

Figure 3 shows an example of the calculations when A = 12m and τ = 12m. A conventional data display where the secondary fields are normalized by the primary field at the receiver and are expressed in parts per million (ppm) is used to present the results. A preliminary examination reveals that the anomaly amplitude is quite sensitive to the keel depth while the anomaly width is related to the keel shape. It should also be noted that the coaxial coil configuration appears to be more sensitive to this quantity than the coplanar one. More work however, is needed to allow for the full interpretation of the AEM anomaly in terms of the keel outline.

Conclusion

The EM modeling problem for a perfectly conductive medium with surface relief can be solved by applying the Neumann boundary condition. The sources of the EM waves may be arbitrary providing that the primary magnetic field at the boundary of the perfect conductor can be calculated. This solution has been confirmed both analytically and by scale model measurements. Using this approach, we have computed the AEM system response for a number of models. The results show that it is feasible to use AEM methods to determine the sea-ice thickness even when it changes rapidly with position. Although the above discussion was restricted to two-dimensional features, the computing technique presented here is readily extended to three-dimensional targets. To demonstrate this point, we present sample data for a "conical" keel in Figure 4.

Acknowledgement

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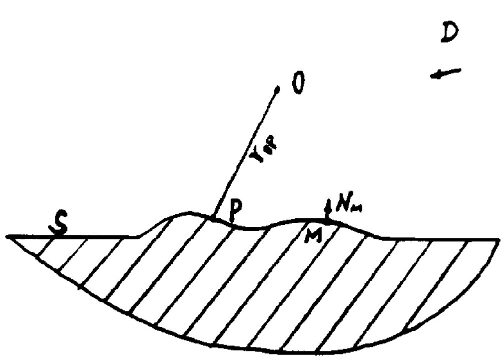

FIG. 1. Draft for exterior Neumann problem.

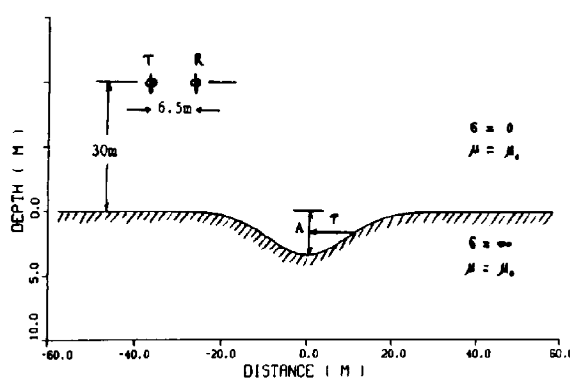

FIG. 2. Cross-section of smooth ice keel (model).

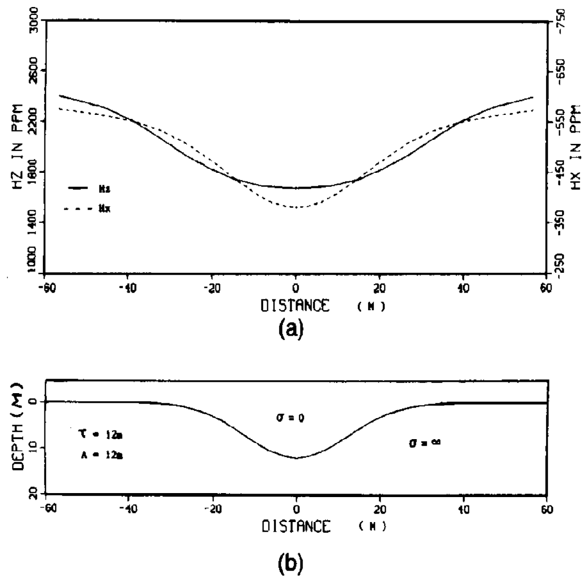

FIG. 3. (a) System responses and (b) cross-section of surface.

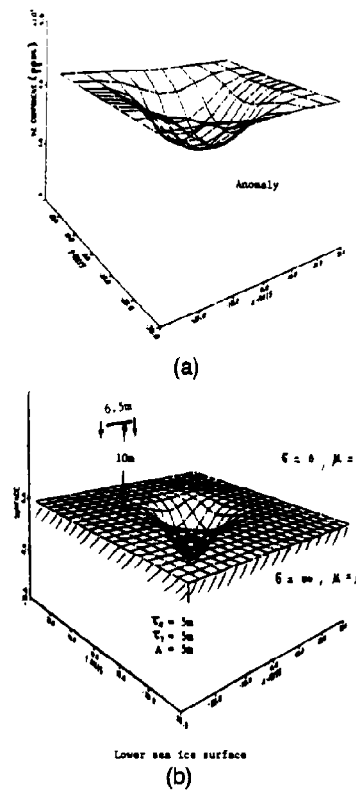

FIG. 4. (a) Vertical dipole system responses; (b) ac magnetic dipole over perfectly conducting medium.